**Bubble Pump Modeling for Solar Hot Water Heater System Design Optimization**

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**Abstract**

Solar thermal water heating systems reduce household energy bills by using the free solar radiation provided by the sun to heat water for residential needs. In order to eliminate the need for electricity to run a pump to drive the fluid circulation in these systems, fluid buoyancy effects can be employed to move the fluid from lower elevations to higher elevations. There are several operational challenges with conventional “thermosyphon” systems, such as reversing flow and overheating, which can all be addressed by using a bubble pump mechanism. Although the solar thermal bubble pump water heating system has been on market, little research has been done on the optimization of the system to improve its efficiency. Engineering Equation Solver software is used to implement a mathematical model for the bubble pump system operation. The model allows for parametric studies of the design attributes to investigate optimum efficiency conditions for the thermally driven pump.

**Nomenclature**

**Symbols**

\[ A \quad \text{Cross-sectional area (m}^2\text{)} \]

\[ A_1 \quad \text{Constant} \]
Bo  Bond Number

$B_1$  Constant

$C$  Constant (with subscripts 1-8)

$C_o$  Distribution parameter

$C_n$  Constants in Chexal-Lellouche Void Model ($n = 1, 2, \ldots 8$)

$COP$  Coefficient of Performance

$D$  Diameter of lift tube (m)

$D_o$  Diameter of entrance tube (m)

$D_2$  Reference diameter (m)

$f$  Friction factor

$f'$  Fanning friction factor

$g$  Acceleration of gravity ($m/s^2$)

$H$  Height of Generator liquid level (m)

$j$  Superficial velocity (m/s)

$K$  Experimental friction relationship

$K_o$  Correlating fitting parameter

$L$  Length of lift tube (m)

$L_o$  Length of entrance tube (m)

$L_C$  Chexal-Lellouche fluid parameter

$m$  Constant (different drift flux analysis than in slug/churn transition analysis)

$m$  Mass flow rate (kg/s)

$n$  Constant

$N_f$  Viscous effects parameter
\[ P \quad \text{Pressure (bars)} \]

\[ Q \quad \text{Volumetric flow rate (m}^3/\text{s)} \]

\[ \dot{Q} \quad \text{Heat transfer rate (W)} \]

\[ r \quad \text{Correlating fitting parameter} \]

\[ Re \quad \text{Reynolds number} \]

\[ S \quad \text{Slip between phases of two-phase flow} \]

\[ T \quad \text{Temperature (K)} \]

\[ V \quad \text{Velocity (m/s)} \]

\[ x \quad \text{Quality} \]

\[ Y \quad \text{Mole fractions} \]

**Greek characters**

\[ \varepsilon \quad \text{Void fraction} \]

\[ \varepsilon_r \quad \text{Pipe roughness (m)} \]

\[ \rho \quad \text{Density (kg/m}^3) \]

\[ \mu \quad \text{Fluid viscosity (kg/m-s)} \]

\[ \sigma \quad \text{Surface tension (N/m)} \]

\[ \Sigma \quad \text{Surface tension number} \]

\[ \dot{\gamma} \quad \text{Volumetric Flow Rate (m}^3/\text{s)} \]

**Subscripts**

\[ 0 \quad \text{State 0 (in governing equations)} \]

\[ 1 \quad \text{State 1 (in governing equations)} \]

\[ 2 \quad \text{State 2 (in governing equations)} \]

\[ a \quad \text{Ammonia} \]
**Introduction**

Renewable energy technologies, such as wind and solar power, are being widely studied by researchers today as many countries are trying to reduce their dependence on non-renewable energy sources (i.e. fossil fuels). Massive use of conventional, non-renewable resources produces greenhouse gases which contribute significantly to climate change. Renewable energy sources, such as solar and wind energy, on the other hand, do not produce greenhouse gases. They are sustainable and free of cost.

According to the World Watch Institute (WWI, 2011), “The United States, with less than 5% of the global population, uses about a quarter of the world’s fossil fuel resources—burning up nearly 25% of the coal, 26% of the oil, and 27% of the world’s natural gas.” In 2009, buildings consumed 72% of electricity and 55% natural gas of total consumption in the US. Water heating for buildings accounted for about 4% of the total energy used in 2009.
Solar thermal water heating (STWH) systems are both cost efficient and energy efficient. The STWH systems are most suitable for places with hot climates and direct sunlight, but can also work quite well in colder climates found throughout the United States. Different designs of the STWH systems have overcome the challenges such as freezing, and overheating. Most STWH systems have a flat plate solar collector tilted towards the south on the roof of the residence as shown in Figure 2. Flat plate solar panels consist of four parts: A transparent cover which allows minimal convection and radiation heat loss, dark color flat plate absorber for maximum heat absorption, pipes which carries heat transfer fluid that remove heat from the absorber, and a heat insulated back to prevent conduction heat loss. Within this collector, there is a network of black tube inside which flows water or some other fluid. The fluid in the tube enters the panel relatively cold and is heated as it flows through the panel as the black exterior of the tubes absorbs the heat from the sun. The heat transfer fluid and the materials selected for the tubes are important parameters for the system to withstand drastic temperature differences of freezing and overheating. If the fluid is water, then it goes right into a hot water storage tank. If another working fluid is used, the heat is then transferred from the working fluid to the water in the hot water tank via a heat exchanger in the tank.
Two of the most common STWH system types are passive and active systems. The fluid in a passive system is driven by natural convection, whereas in active systems, the fluid is moved via a pump. Active systems would be more reliable; however, they require another form of energy input other than solar energy to run the pumps inside the systems. Thus, passive systems are preferred for economic purposes, but they have their own set of technical challenges.

One of the most favorable passive systems is a thermosyphon STWH system, shown in Figure 3. A thermosyphon is an open loop system that can only be used for nonfreezing climates. The simple design of the thermosyphon system is economically efficient. Theoretically, in a thermosyphon system, the tank has to be above the solar panel. Using gravity, the cold water from the tank flows downward and enters the tube beneath the panel, and gets heated up inside the tube. During this process, similar to a hot air balloon, the less dense hot water is buoyant and thus floats to the top of the tube and slowly reaches the top of the tank. Then, more cold water would drain down from the tank and get heated up. The process continues on until the water of the tank reaches thermal equilibrium. Although the system seems ideal, there are problems such as overheating and freezing. Furthermore, the temperature differences of the tank from sunrise to sunset will always be positive; therefore, the pressure difference will also be positive. If the pressure of the system is not designed properly, there will be a reverse-thermosyphon effect, which happens often during drastic temperature drops. The reverse-thermosyphon effect can cause flooding.
A bubble pump, also known as a geyser pump, STWH system, as shown in Figure 4, is an improved version of a closed loop thermosyphon system. It is not an active system, but works as one. The unique design of the system runs like a pump but does not require mechanical work as input to run the pump. When heat is added to the system liquid water changes into two phases, liquid and vapor; which creates a two-phase flow. The buoyance of gas creates a pump-like effect that pushes boiling water to a higher elevation. Unlike the thermosyphon system, the position of the tank does not depend on the solar panel. With an initial hand pump to get the pressure in the bubble tank to be in vacuum, cold water enters to the bottom of the panel. Similar to thermosyphon system, the water heats up through natural convection and hot water rises to the top of the panel due to buoyancy. Hot water leaves the panel through lifting tubes and enters a second reservoir, which then drains down to the top of the tank and creates a pressure difference in the system. Due to the pressure difference, the cold water in the tank will rise up into the panel. When the water in the panel is overheated, the pressure in the system is unstable, then the valve of the header at second reservoir opens, this then is connected to a third reservoir and allows vapor gas to escape the closed system. The third reservoir is connected to the entrance of cold water in the panel. Once the vapor gas condenses, it flows together into the panel again with the cold water. By doing so, it also preheated the cold water entering the panel. The bubble pump system also adds in an antifreeze working fluid to prevent freezing. The most commonly used antifreeze fluid is a mixture of propylene glycol and water. Although the bubble pump is slightly more costly than a thermosyphon system, it does prevent freezing, overheating, as well as reverse-thermosyphon effects.

There are several parameters of the bubble pump which can be adjusted to optimize the STWH system. These parameters include: the diameter of the lift tube, the number of lift tubes, and the material of the lift tube. Slug flow is the optimal operating two phase flow regime for fluid pumping (White, 2001), but because this system will change dynamically with solar input fluctuations, it will be difficult to design the bubble pump to stay in this regime. For a given system operating temperature, previous studies have shown that a smaller than optimal diameter lift tube will experience churn flow and a much higher than optimal diameter tube will produce bubbly flow (White, 2001). As the temperature of the solar flat plate collector changes throughout the day, the two phase flow regime in the lift tube changes between bubbly flow, slug
flow, and churn flow. Because the optimum diameter is limited in size for maximum efficiency, multiple lift tubes may be needed to achieve the desired flow rate of hot water through the system. Multiple lift tubes can help boost up the speed of removing heated water from the flat panel, which then increase the circulation speed of the system and can help the system stay in the slug two-phase flow regime for a longer period of time. Material properties also influence the surface tension related forces in the two phase flow structure which thus influences the performance of the system.

There are very few studies in the literature regarding the operation of a bubble pump driven solar thermal system, however two companies are offering such systems in the U.S. market: Sunnovations and SOL Perpetua. There are geyser pump SHWS patents by Haines(1984) and van Houten(2010) related to these two companies. Both involve copper lifting tubes. Haines (1984) uses a mixture of water and methanol as working fluid of the system, van Houten (2010) uses water, whereas SOL Perpetua (2011) uses water propylene as the working fluid. Li et al (2008) believed the diameter and friction factor of the lifting tube have an inverse relationship to each other, and as diameter of the lifting tube increases, the efficiency of the bubble pump would also increase.

Figure 4: Bubble Pump Flow Schematic
METHODOLOGY

Two-phase flow is the main mechanism for running the bubble pump in a STWH system. As the water boils, the water vapor coalesces and pushes slug of liquid water through the pipe into another water storage reservoir. Although the bubble pump has been used greatly in other products, such as the percolating coffeemaker, rarely any research was done on the optimization of the system (White, 2001). Some key terminology associated with two-phase flow is listed in Table 1. There are four types of basic flow patterns: bubbly, slug, churn, and annular, shown in Figure 5.

Table 1: Two-Phase Flow Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units (SI)</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_G$</td>
<td>kg/m$^3$</td>
<td>Density of gas phase</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>kg/m$^3$</td>
<td>Density of liquid phase</td>
</tr>
<tr>
<td>D</td>
<td>m</td>
<td>Diameter of lift tube</td>
</tr>
<tr>
<td>$A_\text{total} = \pi D^2/4$</td>
<td>m$^2$</td>
<td>Total cross sectional area of pipe</td>
</tr>
<tr>
<td>$A_G$</td>
<td>m$^2$</td>
<td>Cross sectional area gas occupies</td>
</tr>
<tr>
<td>$A_L = A - A_G$</td>
<td>m$^2$</td>
<td>Cross sectional area liquid occupies</td>
</tr>
<tr>
<td>$\varepsilon = A_G/A$</td>
<td>-</td>
<td>Gas void fraction of the flow</td>
</tr>
<tr>
<td>$\dot{V}_G$</td>
<td>m$^3$/s</td>
<td>Gas volumetric flow rate</td>
</tr>
<tr>
<td>$\dot{V}_L$</td>
<td>m$^3$/s</td>
<td>Liquid volumetric flow rate</td>
</tr>
<tr>
<td>$\dot{V} = \dot{V}_L + \dot{V}_G$</td>
<td>m$^3$/s</td>
<td>Total volumetric flow rate</td>
</tr>
<tr>
<td>$j_G = \dot{V}_G/A$</td>
<td>m/s</td>
<td>Gas superficial velocity</td>
</tr>
<tr>
<td>$j_L = \dot{V}_L/A$</td>
<td>m/s</td>
<td>Liquid superficial velocity</td>
</tr>
<tr>
<td>$j = j_G + j_L$</td>
<td>m/s</td>
<td>Total average velocity of flow</td>
</tr>
<tr>
<td>$V_G = j_G/\varepsilon$</td>
<td>m/s</td>
<td>Velocity of the gas</td>
</tr>
<tr>
<td>$V_L = j_L/(1-\varepsilon)$</td>
<td>m/s</td>
<td>Velocity of the liquid</td>
</tr>
<tr>
<td>$\dot{m}_G$</td>
<td>kg/s</td>
<td>Mass flow rate of gas</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>kg/s</td>
<td>Total mass flow rate</td>
</tr>
<tr>
<td>$\dot{x} = \dot{m}_G/\dot{m}$</td>
<td>-</td>
<td>Quality</td>
</tr>
<tr>
<td>S = $V_G/V_L$</td>
<td>-</td>
<td>Slip between phases</td>
</tr>
</tbody>
</table>
A series of momentum and mass flow balances can be performed to model the operation of the bubble pump system based on the definitions of the state locations in Figure 6. These equations are provided in detail as follows:

Momentum equation from $P_{\text{sys}}$ to 0 gives:
\[ P_0 = P_{sys} + \rho_L g H - \rho_L \frac{V_0^2}{2} \quad \text{[1]} \]

Where:

- \( V_0 \) is the velocity (m/s) at point 0 (liquid solution)

Momentum equation from 0 to 1 yields (including pressure drop from friction):

\[ P_1 = P_0 - \rho_L V_0 (V_1 - V_0) - \rho_{TP} g H_{FP} \quad \text{[2]} \]

Where:

- \( V_1 \) is the velocity (m/s) at state 1
- \( D_0 \) is the diameter (m) of the water entrance line
- \( \rho_L \) is the density (kg/m\(^3\)) of the water entrance line
- \( \rho_{TP} \) is the density (kg/m\(^3\)) of the two phase mixture in flat panel
- \( H_{FP} \) is the height (m) of the water level in flat panel

Conservation of mass from state 0 to 1 yields:

\[ \rho_L A_0 V_0 = \rho_L A_0 V_1 \quad \text{[3]} \]

Where:

- Area of the water entrance line (m\(^2\)): \( A_0 = \pi D_0^2 / 4 \quad \text{[4]} \)

Therefore:

\[ V_0 = V_1 \quad \text{[5]} \]

Conservation of momentum from state 1 to 2, neglecting friction in this transition:

\[ P_2 = P_1 - \rho_{TP} V_1 (V_2 - V_1) \quad \text{[6]} \]

where \( \rho_{TP} \) is the homogeneous density of the two-phase flow. Since the velocities of each phase in the region between 1 and 2 are approximately equal (slip, \( S=1 \)) a homogeneous density is used in this momentum equation. This expression for the homogeneous density can be found from the conservation of mass from 0 to 1.

Conservation of mass from state 0 to 1 for the system yields:

\[ \rho_L A_0 V_0 = \rho_{TP} A_0 V_1 \quad \text{[7]} \]
The homogeneous density follows from this equation after substituting for the Areas at state 0 and state 1:

$$\rho_{TP} = \frac{\rho_L A_0^2 V_0}{A_1^2 V_1} \quad [8]$$

At this point, the two-phase flow terminology from Table 1 is needed to proceed because the flow in the lift tube is most clearly defined in these terms. The definitions of superficial velocities and void fraction can be related to the terminology used thus far.

Since states 0 and 1 are under liquid conditions:

$$V_0 = \frac{Q_L}{A_0} \quad [9]$$

While the definition of the superficial liquid velocity, $j_L$ is:

$$j_L = \frac{Q_L}{A} \quad [10]$$

Therefore:

$$V_1 = \frac{Q}{A_1} \quad [11]$$

Additionally, state 2 has two phases, but $V_2$ still describes the total average velocity of the mixture:

$$V_2 = \frac{Q_L + Q_g}{A} = \frac{Q}{A_2} \quad [12]$$

This is precisely the definition of $j$. Therefore:

$$V_2 = j \quad [13]$$

It follows from equations [11] and [13] that:

$$V_2 - V_1 = j - j_L \left( \frac{A}{A_0} \right) \quad [14]$$

Also, the void fraction is defined as the average cross sectional area occupied by the gas divided by the total cross sectional area of the pipe.

Therefore:
\[ \frac{A_L}{A_2} = 1 - \varepsilon \quad \text{(15)} \]

Now the momentum equation in the lift tube (from state 2 to \( P_{\text{sys}} \)) can be stated as:

\[ P_2 = P_{\text{sys}} + f_{\text{TP}} \left( \rho_L j_L + \rho_G j_G \right) \left( \frac{L}{D} \right) + \rho_L L g (1 - \varepsilon) \quad \text{(16)} \]

where \( f_{\text{TP}} \) is the two-phase friction factor, based on average properties of liquid and gas and \( \rho_{\text{TP}} \) is the two-phase density of the fluid mixture in the lift tube. Here, for the frictional pressure drop term, a two-phase density is required instead of a homogeneous one since there is now slip between the two phases. This two-phase density can be found from the density definition applied to the lift tube volume:

\[ \rho_{\text{TP}} = \rho_G \varepsilon + \rho_L (1 - \varepsilon) \quad \text{(17)} \]

Therefore, combining Equations [1], [2], [5], [6] and [16], a general equation for the submergence ratio (H/L), which describes the average pressure gradient along the lift tube, can be solved as:

\[ \frac{H}{L} = \frac{f_{\text{TP}} \left( \rho_L j_L + \rho_G j_G \right)^2}{2 g D \rho_L \rho_{\text{TP}}} + \frac{j_L \left( \frac{D}{D_0} \right)^4}{2 g L} + \frac{j_L \rho_{\text{TP}} \left( \frac{D}{D_0} \right)^2 \left( j - j_L \left( \frac{D}{D_0} \right)^2 \right)}{\rho_L g L} + (1 - \varepsilon) \quad \text{(18)} \]

**THE DRIFT FLUX MODEL**

The drift flux model is now the widely accepted method for analyzing void fractions in two-phase flow. This method, formalized by Zuber and Findlay in 1965, provides a means to account for the effects of the local relative velocity between the phases as well as the effects of non-uniform phase velocity and concentration distributions.

While many others contributed to the beginnings of two-phase flow theory, Zuber and Findlay’s (1965) analysis establishes the basis of the drift flux formulation used today (Chexal 1997). It relates the average gas void fraction of the two-phase flow to: 1) the superficial velocities (the velocity each phase would have if they occupied the entire area of the pipe alone) of the gas and liquid phases; 2) \( C_\alpha \), the distribution parameter; and 3) \( V_{\text{sl}} (= V_G - j) \), the drift velocity. The resulting drift flux model can be summarized by the following equation:

\[ \varepsilon = \frac{j_G}{C_\alpha (j_L + j_G) + V_{\text{sl}}} \quad \text{(19)} \]
Many authors have formulated empirical correlations for $C_0$ and $V_{gj}$ depending on the two-phase vertical flow regimes shown in Figure 5 and other parametric effects. In the current study, the diameter of the lift tube and surface tension are two important parameters of the bubble pump design that could potentially increase the efficiency of the pump. Therefore, the correlation of de Cachard and Delhaye (1996), which took surface tension effects into account, is used for the slug flow regime. The de Cachard and Delhaye empirical correlations for $V_{gj}$ and $C_0$ are:

$$V_{gj} = 0.345 \left( 1 - e^{-0.01N_f/0.345} \right) \left( 1 - e^{(3.37 - Bo)/m} \right) g D$$

$$C_0 = 1.2$$

Where:

$$\left( N_f \right)^2 = \frac{\rho_L (\rho_L - \rho_G) g D^3}{\mu_L^2}$$

Bond number:

$$Bo = \frac{(\rho_L - \rho_G) g D^2}{\sigma}$$

And $m$ is defined for different ranges of $N_f$:

$N_f > 250$:  
$$m = 10$$  \[23a\]

$18 < N_f < 250$:  
$$m = 69 (N_f)^{0.35}$$  \[23b\]

$N_f < 18$:  
$$m = 25$$  \[23c\]

It is expected that the system will also experience bubbly and churn flow regimes, thus correlations are needed to define the transitions between bubbly, slug and churn along with correlations for the drift flux velocity and $C_0$ for the bubbly and churn flow cases.

The transitions between flow regimes are often determined by void fraction $\varepsilon$ (Taitel and Bornea, 1980). Taitel and Bornea (1974) found that in bubbly flow $\varepsilon < 0.25$, $\varepsilon = 0.25$ at slug flow regime, and $\varepsilon > 0.25$ during churn flow. Although for most cases bubbly flow happens when the void fraction is less than 0.25, the flow pattern varies when diameter of the lift tube changes. Thus, Hasan’s (1987) correlation was used in the study for transition between bubbly flow and slug flow. Hasan defines the relative velocity of the vapor bubbles, $U_0$, and the larger coalesced Taylor bubble, $U_G$, in the flow as follows:
\[ U_o = 1.53 \left[ \frac{g \rho_L - \rho_g \sigma}{\rho_L^3} \right]^2 \tag{24a} \]
\[ U_G \approx 0.35 \frac{gD}{\overline{gD}} \tag{24b} \]

Bubbly flow then occurs when the Taylor bubble outruns the gas bubbles and thus the smaller bubble: or \( U_o < U_G \). In this case the correlations for bubbly flow are:

\[ C_0 = 1.2 \]
\[ \overline{V_{gj}} = 0.43 \frac{U_G}{(j_G - 0.36 \times U_G)} \tag{25} \]

Slug flow occurs when the vapor bubbles run into the back of the forming Taylor bubble, thus \( U_o > U_G \), and then the previous correlations for \( C_0 \) and \( V_{gj} \) can be used.

For the transition from slug flow to churn, the procedure from Hewitt (1965) was followed. Hewitt found that the transition between slug flow and churn flow could be predicted for a particular system using the following set of equations:

\[ \theta = \overline{j_{bx}} + 0.96 \overline{j_{fx}} \tag{26} \]

Where

\[ j_{bx} = j_b \frac{\rho_g}{gD(\rho_l - \rho_g)} \quad \text{and} \quad j_{fx} = j_f \frac{\rho_l}{gD(\rho_l - \rho_g)} \tag{27} \]

White (2001) found that slug-churn transition occurred at a value of 0.83 for \( \theta \) in a similar bubble pump system. The equations then used for churn flow were (Hasan,1987):

\[ \theta > 0.83 \]
\[ C_0 = 1.15 \]
\[ \overline{V_{gj}} = 0.345 \frac{g * D * (\rho_L - \rho_G) / \rho_L}{\overline{gD}} \tag{28} \]

The working fluid used in the current study is water. The SHWS modeled runs under a vacuum pressure of 20-25 inches of mercury. While the model of the solar thermal water heating systems require a solar flat plate panel collector to collect solar energy and heat up water in the panel, it is assumed that the panel is about 50% efficient and can collect a theoretical heat input of 700W/m\(^2\) from a standard 3 m\(^2\) panel, and the temperature of the fluid entering the lift tube is at the saturated temperature under these pressure and heat flux conditions.

The efficiency of the solar thermal system is measured by the mass flow rate of the hot water output of the bubble pump as the input to the system is free solar thermal energy. The minimum hot water mass flow rate output requirement from the bubble pump is 1 liter per minute for two lift tubes.
RESULTS & DISCUSSION
To solve all of the non-linear equations, Engineering Equation Solver (EES) software is used. EES is an equation solver software that can iteratively solve thousands of linear or non-linear equations simultaneously. It has built in libraries for thermodynamic and fluid transport properties, thus it is widely used in the Mechanical Engineering field. It can also solve differential and integral equations, which is helpful for the optimization of parameters for the solar thermal bubble pump water heating systems.

Figure 7: Liquid Mass Flow Rate Versus Lift Tube Diameter for Varying System Pressure Conditions.
Figure 8a and b: Liquid Mass Flow Rate Versus Heat Input to the Bubble Pump for Different Lift Tube Diameters and System Pressure Conditions.
Figures 7 through 9 show the results of parametric studies for the bubble pump design for integration with a solar thermal hot water heating system. As shown in Figure 7, as the diameter for a bubble pump design increases, the mass flow rate of the liquid being pumped through the system increases to a peak value and then decreases again. As this occurs, the two phase flow regime changes from churn to slug and lastly to bubbly flow. With a lift tube 1 cm in diameter, the system is operating in churn flow at an average solar energy input of 700 W/m² and the output flow rate is 1.8 liters per hour. A lift tube of diameter of 3 cm will result in the highest hot water mass flow rate output of about 7 liter per hour. However, it is unrealistic to have such high solar energy input throughout the day everyday. Figure 8 shows the variation in liquid mass flow rate or a range of heat transfer rates into the system. As the diameter increases more, the two phase flow regime becomes bubbly flow. Thus, having a lift tube of just less than 3 cm, for instance 1 inch in diameter, will likely produce more consistent output from the bubble pump SHWS throughout the year while receiving the benefit of an output flow rate of 6 to 6.5 liter per hour. As the temperature in the day fluctuates, the pressure inside the system also changes; however, the pressure in the system can be controlled by the valve in the header where the vapor collects after the pump. As shown in Figure 9, as the pressure of the system becomes a stronger vacuum, the saturated temperature for water decreases, and the system will require less solar energy input to pump water. As the pressure increased to 0.45 bars, the mass flow rate increased slightly with a smaller diameter lift tube, whereas, the larger diameter lift tubes’ mass flow rate decreased compared to the 0.366 bar case. On the other hand, the mass flow rate decreases as pressure increases at a lower solar energy heat input. With a pressure of 0.25 bars, the output mass flow rate of the system also provides about 6.5 liter per hour of hot water output at an average solar energy input. Although pressure is an important parameter of the system, with the optimized tube diameter, the pressure of the system has less effect on the system. As temperature...
fluctuates throughout the day, the hot water mass flow rate output also varies. At a constant
pressure of 0.25 bars of the system, and lift tube diameter of 1 inch, it is found that the maximum
mass output flow rate occurs at 800W/m$^2$, which is consistent with the other results of at least 6
liter per hour of hot water output.

**CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK**

The model results showed that having a 1 inch lift tube operating at a pressure of 0.25 bars can
boost up the mass flow rate output to 6 liter per hour, which is about 6 times the required output.
Further study can be done to more effectively simulate the system by adding in models of the
solar panel, the time varying impacts of the solar insulation fluctuation, and additional lift tubes
to study the detailed optimization of the bubble pump SHWS. Experimental performance should
be compared with the resulting model before further analysis and implementation of system
changes.


